In this lecture, Dr. Shen went over the problem assigned in class on 12 April, and also the problems in the second test. Dr. Shen explained the first problem, and students were called to answer the test problems at the board.

The equations we need to solve the first problem are for the reduced mass:

$$\mu = \frac{m1 \cdot m2}{m1 + m2}$$

and the equation for the energy levels:

$$E_{0,1,0,0} = -\frac{Z'^2 \Re}{n^2} \xrightarrow[n=1 \text{ at ground state}]{} Z'^2 \frac{\mu}{m_{\text{electron}}} \Re_{\text{Hydrogen}} = \left(\frac{Z}{\varepsilon}\right)^2 \frac{\mu}{m_{\text{electron}}} \Re_{\text{Hydrogen}}$$

where $\Re_{\text{Hydrogen}} = 13.6 \text{ eV}$.

Let's continue to solve the problem that we had last week. For the first one, a deuteron and an electron, a deuteron is a neutron and a proton. The mass of a neutron is about the same as the mass of a proton. So the deuteron has a reduced mass of:

$$\mu_{\text{deuteron}} = \frac{2m_{\text{proton}} \cdot m_{\text{electron}}}{2m_{\text{proton}} + m_{\text{electron}}} \xrightarrow[m_{\text{proton}} \gg m_{\text{electron}}]{} \frac{2\cancel{m_{\text{proton}}} \cdot m_{\text{electron}}}{2\cancel{m_{\text{proton}}}} = m_{\text{electron}}$$

The deuteron has a charge of +e. Therefore the energy to separate the electron from the deuteron is:

$$E_{\text{deuteron}} = Z'^2 \Re_{\text{Hydrogen}} \frac{\mu_{\text{deuteron}}}{m_{\text{electron}}} = -(1)^2 (13.6 \text{ eV}) \frac{1}{1} = -13.6 \text{ eV}$$

It has a positive charge the magnitude of an electron charge.

The second one, He+, generally inside the nucleus, you have 2 protons and 2 neutrons. So the reduced mass is:

$$\mu_{\text{He}^+} = \frac{4m_{\text{proton}} \cdot m_{\text{electron}}}{4m_{\text{proton}} + m_{\text{electron}}} \xrightarrow[m_{\text{proton}} \gg m_{\text{electron}}]{} \frac{4\cancel{m_{\text{proton}}} \cdot m_{\text{electron}}}{4\cancel{m_{\text{proton}}}} = m_{\text{electron}}$$

The charge is +2e, so Z'=2. Therefore:

$$E_{\text{He}^+} = Z'^2 \Re_{\text{Hydrogen}} \frac{\mu_{\text{He}^+}}{m_{\text{electron}}} = -(2)^2 (13.6 \text{ eV}) \frac{1}{1} = -54.4 \text{ eV}$$

Helium binding energy is 4 times larger than Hydrogen.

The next problem, the positron has the mass of an electron. So that

$$\mu_{\text{Positronium}} = \frac{m_{\text{electron}} \cdot m_{\text{electron}}}{m_{\text{electron}} + m_{\text{electron}}} = \frac{m_{\text{electron}} \cdot \cancel{m_{\text{electron}}}}{2\ \cancel{m_{\text{electron}}}} = \frac{m_{\text{electron}}}{2}$$

The positron

$$E_{\text{Positronium}} = Z'^2 \Re_{\text{Hydrogen}} \frac{\mu_{\text{Positronium}}}{m_{\text{electron}}} = -(1)^2 (13.6\text{ eV}) \frac{1}{2} = -6.8\text{ eV}$$

The last problem, the exciton, is a system of an electron and a hole. The mass of the hole, $m_h = 2m_0$, the mass of the electron is $m_0$. The reduce mass of the exciton is:

$$\mu_{\text{exciton}} = \frac{2m_0 \cdot m_0}{2m_0 + m_0} = \frac{2m_0 \cdot \cancel{m_0}}{3\ \cancel{m_0}} = \frac{2m_{\text{electron}}}{3} = \frac{2m_{\text{electron}}}{3}$$

The charge of the hole is +e, but it is screened by a dielectric constant of 10. Therefore the Z' = 1/10.

$$E_{\text{Exciton}} = Z'^2 \Re_{\text{Hydrogen}} \frac{\mu_{\text{Exciton}}}{m_{\text{electron}}} = -\left(\frac{1}{10}\right)^2 (13.6\text{ eV}) \frac{2}{3} = -90.7\text{ meV}$$

For example, the binding energy of ZnO is approximately -64 meV. The exciton exists because room temperature is approximately 20 meV. But for the superlattice (quantum well) of GaAs, the binding energy is about 10 meV. So an exciton can't exist in GaAs because room temperature the temperature will thermalize it and it becomes free. If you can form an exciton laser if the exciton can exist. For ZnO crystal laser, you can use a 355 nm optical pump to got high density exiton. Then you get 390 nm laser line emissions from ZnO. The problem is impurity. If you got good n-type and p-type becomes good laser. I hope you can solve this problem.

Now we are going to solve the problems in the second test. Meg was asked to solve problem #2. At the board she wrote:

The Hamiltonian for a 2-D harmonic oscillator:

$$H = \frac{p^2}{2m} + V(r) = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{m\omega^2 x^2}{2} + \frac{m\omega^2 y^2}{2} = H_x + H_y$$

The Schroedinger Equation is: $\hat{H}\varphi = E\varphi \rightarrow \left(H_x + H_y\right)\varphi = \left(E_x + E_y\right)\varphi$

Since the Hamiltonian's commute, separation of variables can be used to express $\varphi$ as:

$\varphi(x,y,z) = X(x)Y(y)$

Each variable is a 1-D Harmonic oscillator

$$\left(-\frac{\hbar^2}{2m}\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} + \frac{m\omega^2 x^2}{2}\right) = E_x \quad \text{and} \quad \left(-\frac{\hbar^2}{2m}\frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} + \frac{m\omega^2 y^2}{2}\right) = E_y$$

Define: $\beta^2 \equiv \frac{m\omega}{\hbar}$

The eigenstates and eigenenergies of the Hamiltonians $H_x$ and $H_y$ are:

$$X_{n_x}(x) = A_{n_x}\mathcal{H}_{n_x}(\beta x)e^{-\beta^2 x^2/2} \qquad E_{n_x} = \hbar\omega\left(n_x + \tfrac{1}{2}\right) \qquad n_x = 0,1,2,3,\cdots$$

Where $\mathcal{H}$ are the $n^{th}$ order Hermite polynomials and $A_n$ are the normalization constants.

The same result is true for Y(y) and the eigenfunction is their product.

The eigenenergy is: $E_{n_x n_y} = E_{n_x} + E_{n_y} = \hbar\omega\left(n_x + n_y + 1\right) = \hbar\omega\left(n+1\right)$

The degeneracy is found by looking at the number of states – number of ways you can add two integers to get a third – it is *D(n)=n+1*.

The second, problem #4, problem William (I think that is his name) was asked to solve. He solved it this way:

$$P_n = \frac{\left|c_n\right|^2}{\sum_j^{nstates}\left|c_j\right|}$$

$$\Phi\left(\theta,\varphi\right) = \frac{5Y_1^1 + 3Y_5^1 + 2Y_5^{-1}}{\sqrt{38}}$$

for $l = 5$ there are actually 2 states so that:

$$P_n = \frac{\left|c_n\right|^2}{\sum_j^{nstates}\left|c_j\right|} = \frac{\left|\frac{3}{\sqrt{38}}\right|^2 + \left|\frac{2}{\sqrt{38}}\right|^2}{\left|\frac{5}{\sqrt{38}}\right|^2 + \left|\frac{3}{\sqrt{38}}\right|^2 + \left|\frac{2}{\sqrt{38}}\right|^2} = \frac{13}{38}$$

Scott Feinstein solved the 5th problem. We want to find the expectation values <Lx> and <Lx$^2$>. We have the operators:

Lz=$\hbar \boldsymbol{m}$ and L=$\hbar^2 \boldsymbol{l}(\boldsymbol{l}+1)$.

You start with:

$$\overbrace{\langle l,m|\hat{L}_x|l,m\rangle}^{\text{Start with}} = \overbrace{\langle l,m|\tfrac{1}{2}\left(\hat{L}_+ + \hat{L}_-\right)|l,m\rangle}^{\text{Use ladder operators}} = \overbrace{\tfrac{1}{2}\langle l,m|...|l,m+1\rangle + \tfrac{1}{2}\langle l,m|...|l,m-1\rangle}^{\text{expand out, inside is not important because they are orthogonal}} = 0$$

Now we want to find <Lx$^2$>.

$$L^2 = L_x^2 + L_y^2 + L_z^2 \Rightarrow L_x^2 + L_y^2 = L^2 - L_z^2$$

$$\begin{aligned}
\langle l,m|\hat{L}_x{}^2|l,m\rangle &= \langle l,m|\left(\tfrac{1}{4}\left(\hat{L}_+ + \hat{L}_-\right)\right)^2|l,m\rangle \\
&= \tfrac{1}{4}\langle l,m|\left(\hat{L}_+\hat{L}_+ + \hat{L}_+\hat{L}_- + \hat{L}_-\hat{L}_+ + \hat{L}_-\hat{L}_-\right)|l,m\rangle \\
&= \tfrac{1}{4}\langle l,m|\hat{L}_+\hat{L}_- + \hat{L}_-\hat{L}_+|l,m\rangle \\
\langle l,m|\hat{L}_y{}^2|l,m\rangle &= \langle l,m|\left(\tfrac{1}{4}\left(\hat{L}_- - \hat{L}_+\right)\right)^2|l,m\rangle \\
&= \tfrac{1}{4}\langle l,m|\left(\hat{L}_+\hat{L}_+ - \hat{L}_+\hat{L}_- - \hat{L}_-\hat{L}_+ + \hat{L}_-\hat{L}_-\right)|l,m\rangle \\
&= \tfrac{1}{4}\langle l,m|\hat{L}_+\hat{L}_- + \hat{L}_-\hat{L}_+|l,m\rangle \\
&= \langle l,m|\hat{L}_x{}^2|l,m\rangle
\end{aligned}$$

$$L_x^2 + L_y^2 = L_x^2 = L^2 - L_z^2 = \tfrac{1}{2}\hbar^2\left\{l(l+1) - m^2\right\}$$

Nicholas Mosher solved problem #3. Here:

$$I = 2ma^2$$

$$\hat{H} = \frac{\hat{L}^2}{2I} = \frac{\hat{L}^2}{4ma^2}$$

$$E_l = \frac{\hbar^2 l\left(l+1\right)}{4ma^2} \approx \frac{\hbar^2}{4ma^2}$$

$$\left(13.6\text{ eV}\right)\frac{m_e}{m_p} \approx 10\text{ meV}=10^{-2}\text{ eV}$$

No homework was assigned. Next lecture we will talk about transition rules.